Local limits of high genus triangulations

Baptiste Louf (IRIF Paris Diderot)

joint work with Thomas Budzinski

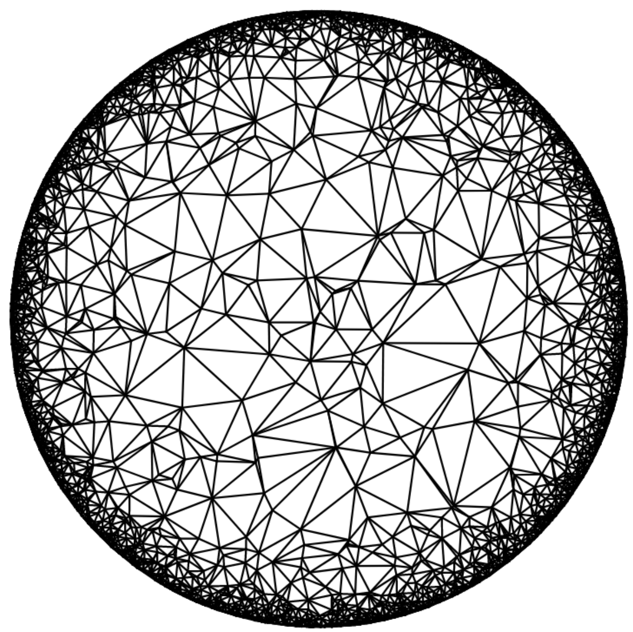
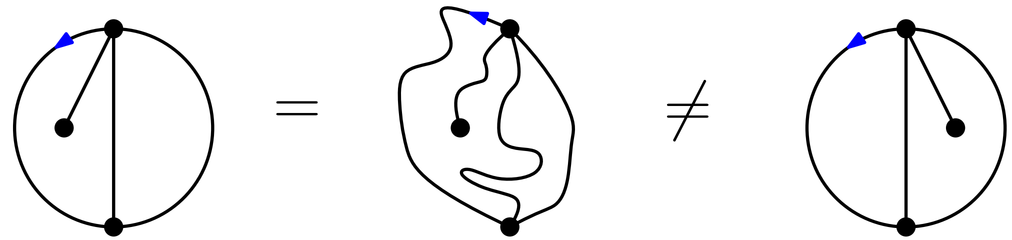


image : N. Curien

Work supported by the grant ERC – Stg 716083 – “CombiTop”

Maps and triangulations

Map = embedding up to homeomorphism of a connected multigraph (loops and multiple edges allowed) in a compact connected orientable surface.
Rooted = an oriented edge is distinguished



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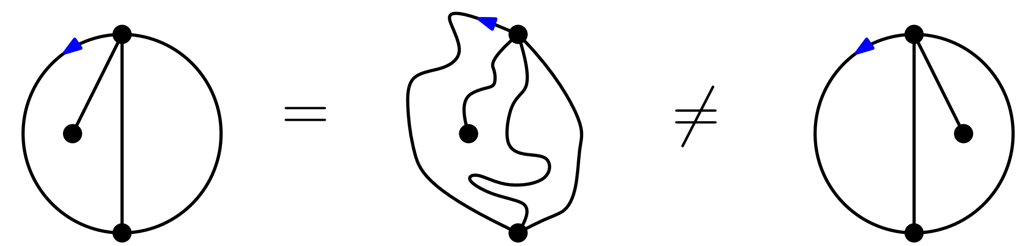
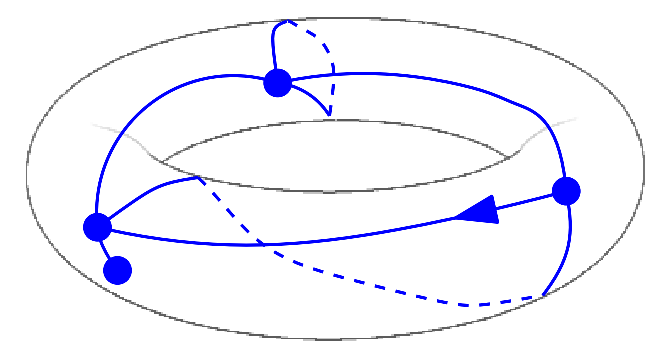


Genus g of the map $=$ genus of the surface $= \#$ of handles



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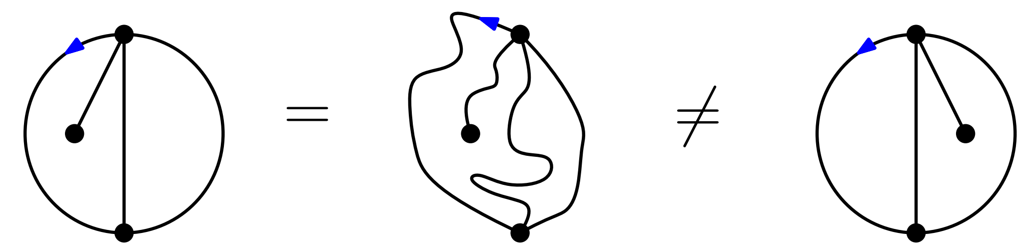

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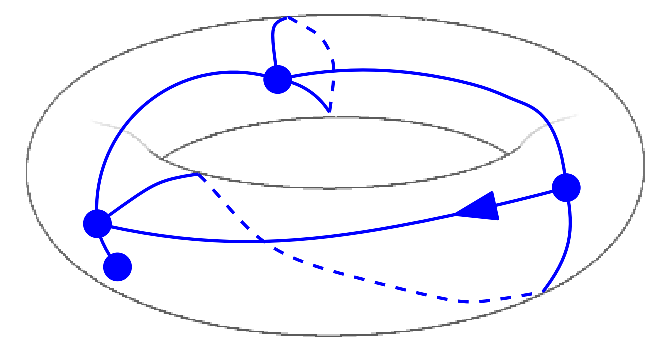
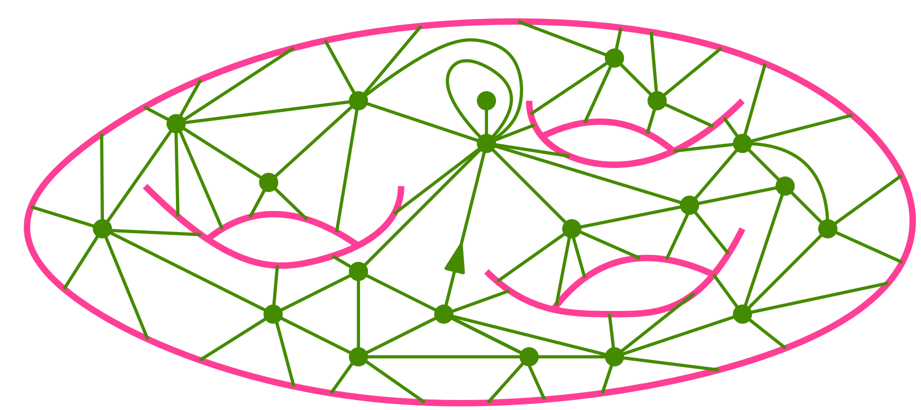


Triangulation = all faces of degree 3



What does a triangulation look like around the root ?

How similar are two triangulations locally ?

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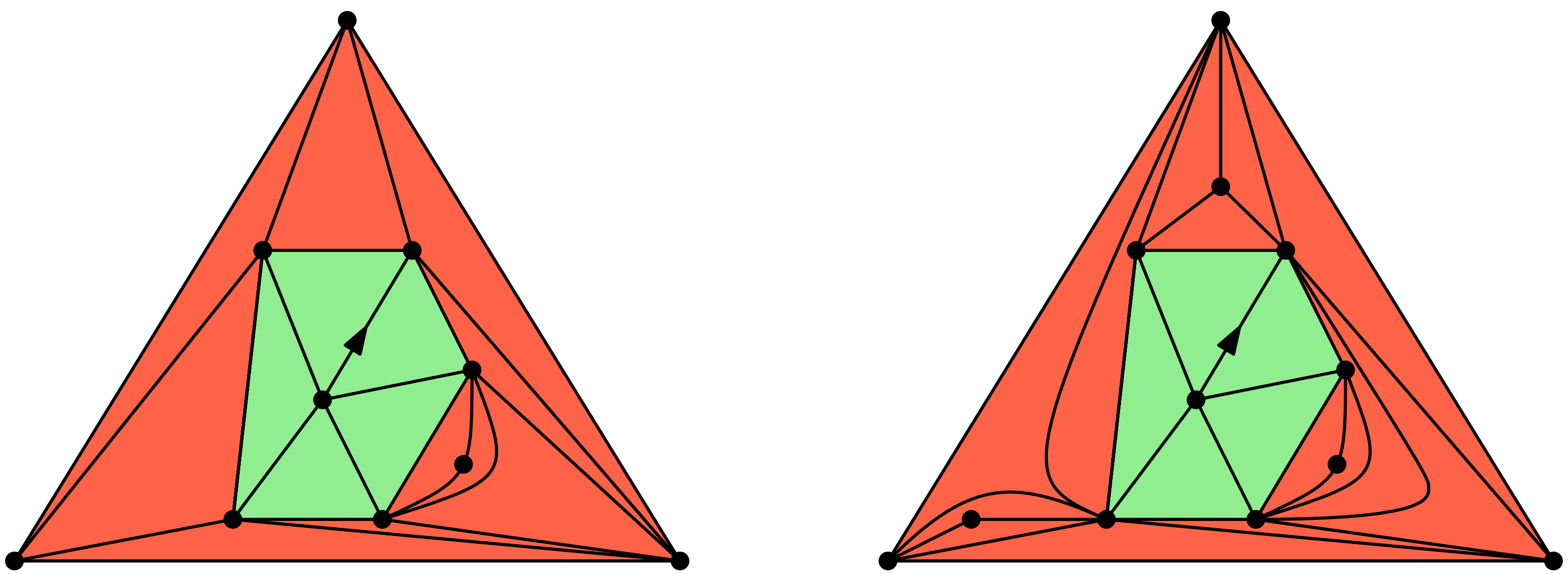
The local distance :
$d_{loc}(T, T') = (1 + \sup\{r | B_r(T) = B_r(T')\})^{-1}$



What does a (large, random) triangulation look like around the root ?

The limit (in law) w.r.t d_{loc} is called the local limit.

Question : let (T_n) be a sequence of random triangulations, whose size $\to \infty$, is there a local limit ? What does it look like ?

[Angel, Schramm '02] : uniform planar triangulations converge to an infinite triangulation called the Uniform Infinite Planar Triangulation (UIPT).

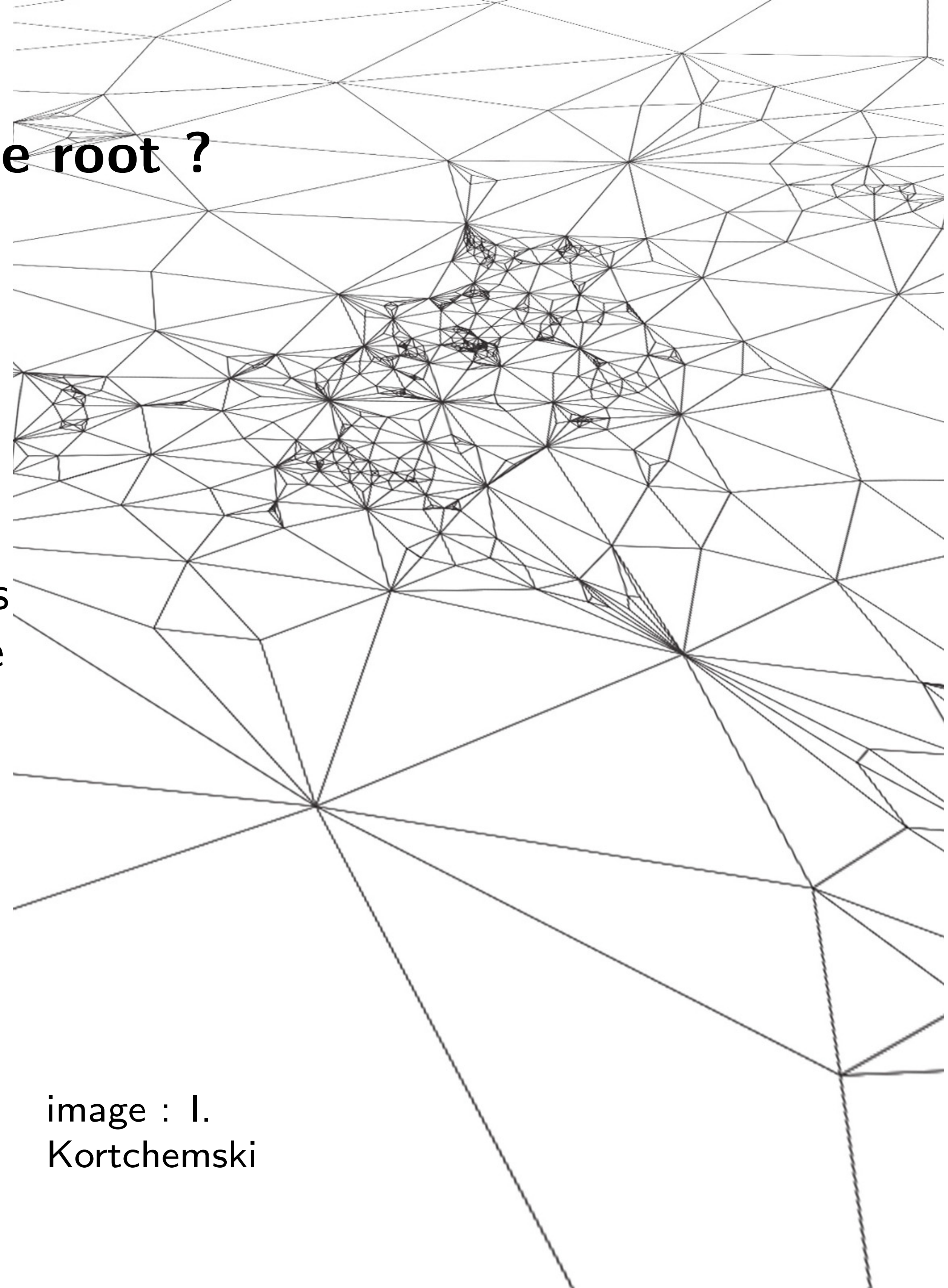
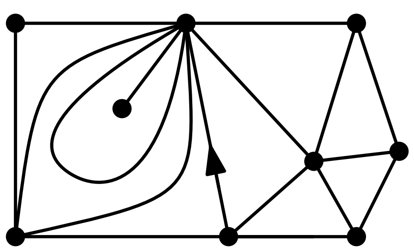


image : I. Kortchemski

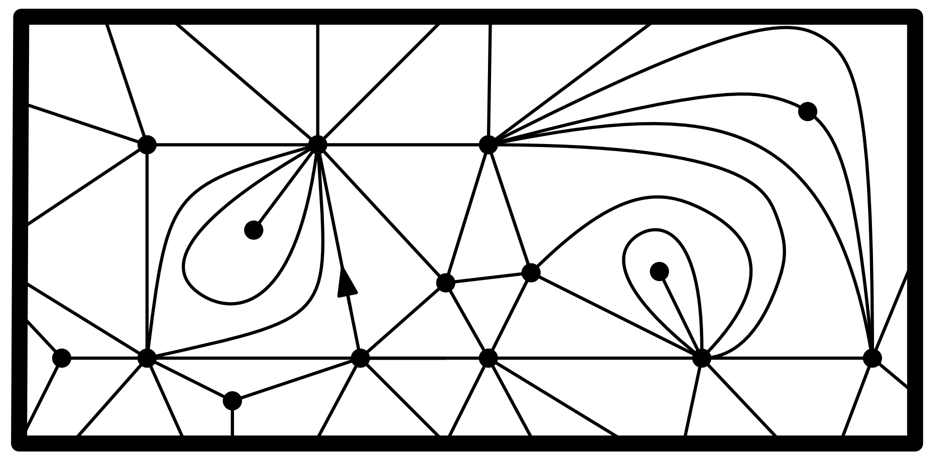
Properties of the UIPT

Spatial Markov property : $\mathbb{P}(t \subset \mathbb{T}) = C_p \lambda_c^{|v|}$



$p = 7, \ |v| = 9$

$\subset$



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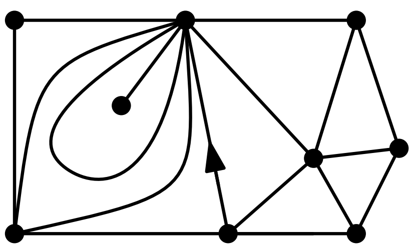

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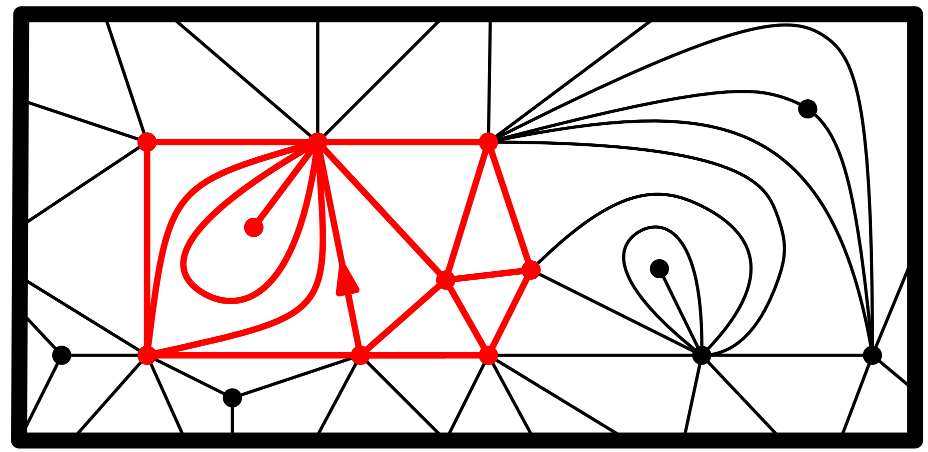

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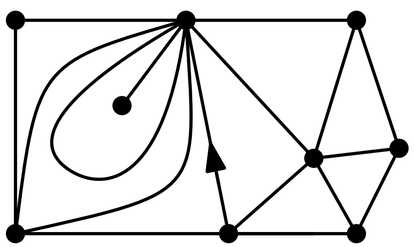

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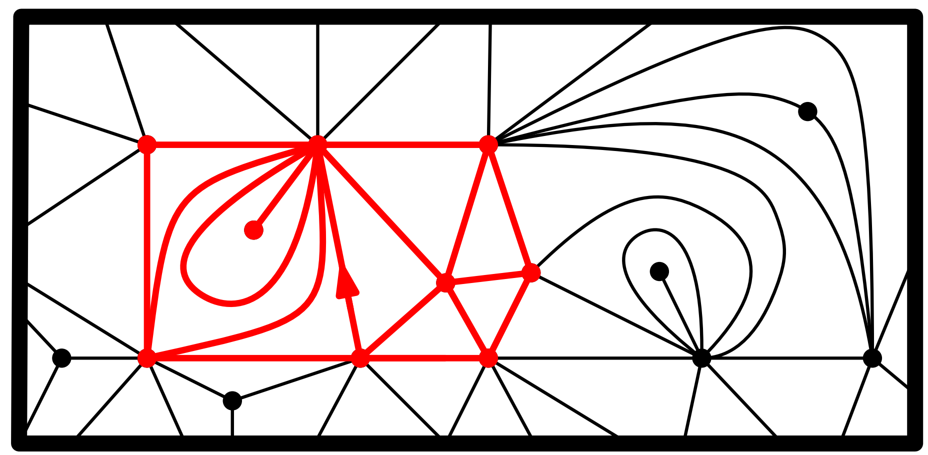

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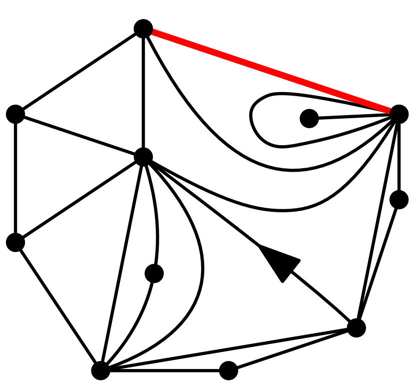
Peeling process : Discover $\mathbb{T}$ step by step, unveil triangles.



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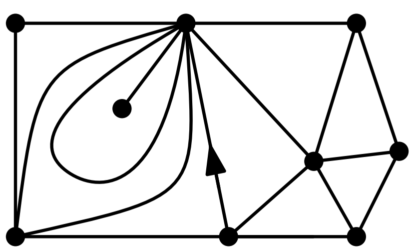

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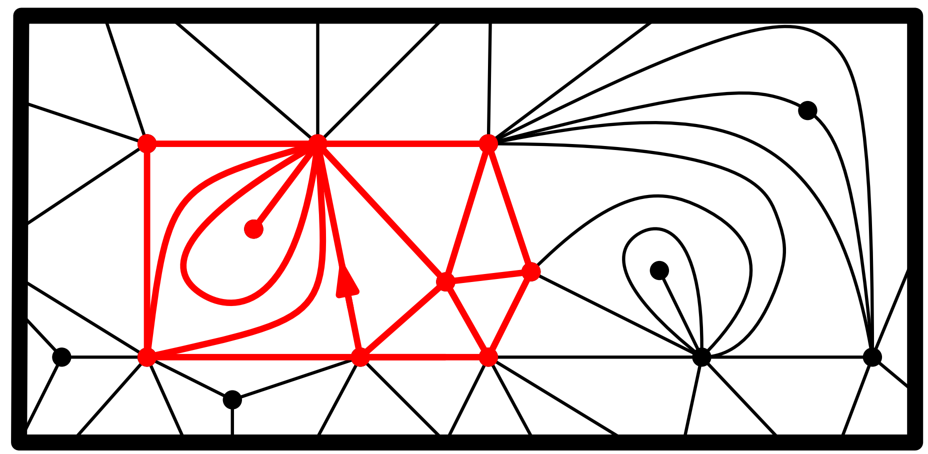

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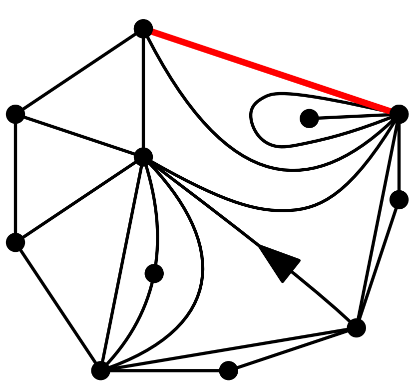

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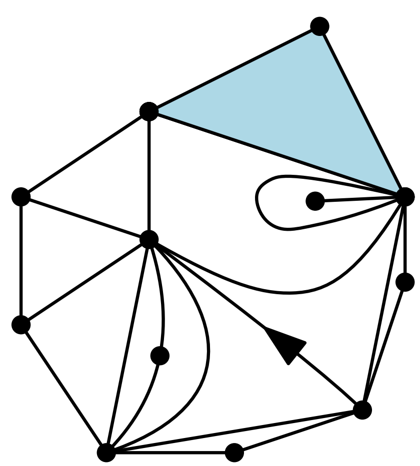

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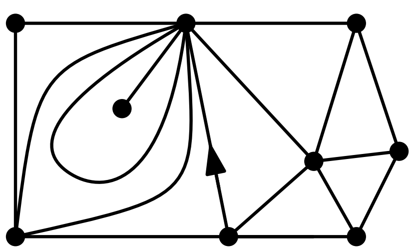

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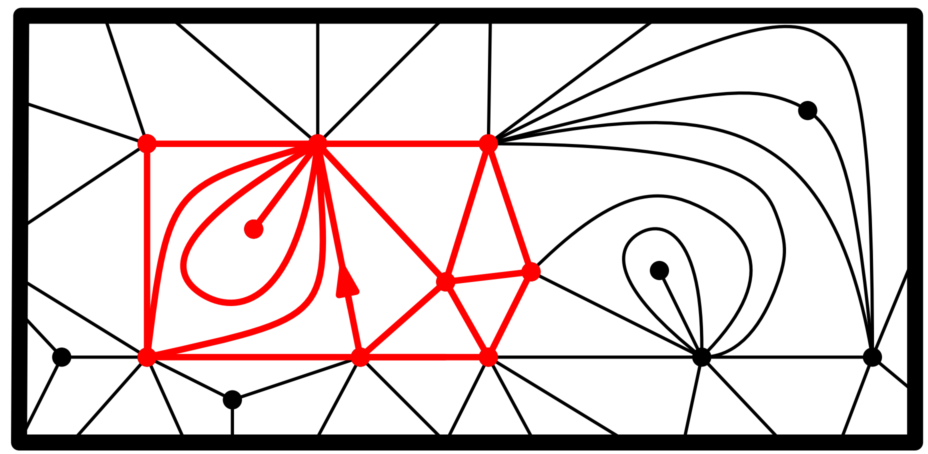

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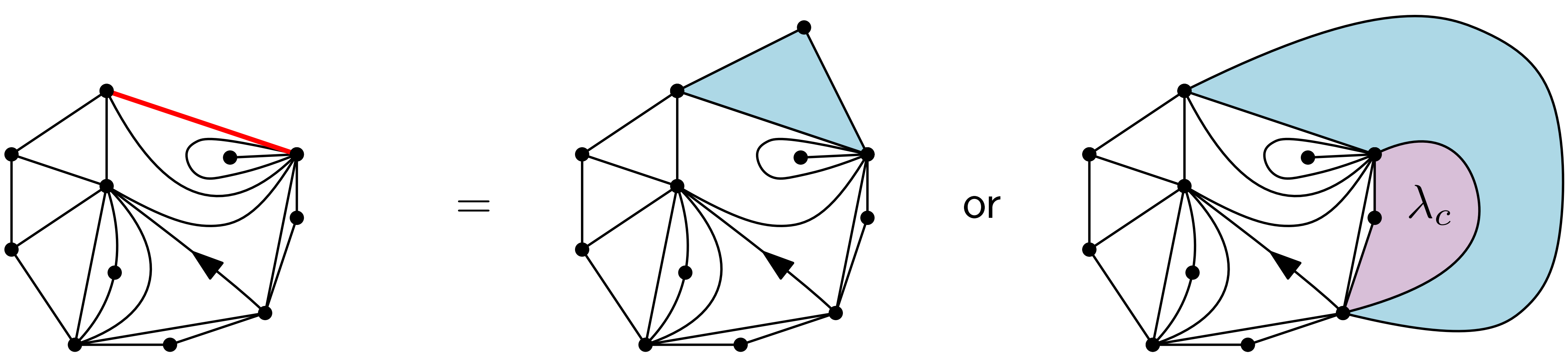


(x2)

The PSHIT

Introduced by Curien in 2012

Defined in the same way as the UIPT, by with $\lambda \in]0, \lambda_c]$.

For $\lambda < \lambda_c$, has an hyperbolic flavour : the "average degree" of a vertex is higher than 6 (the value in a regular planar triangulation), the balls have exponential growth, ...

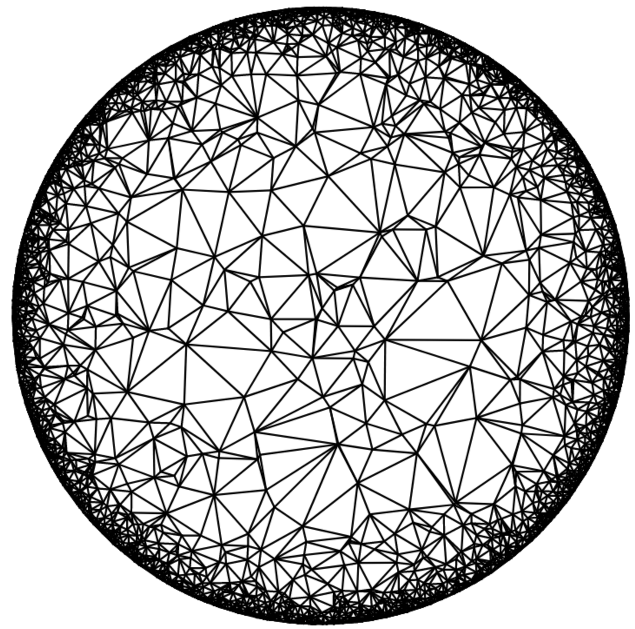


image : N. Curien

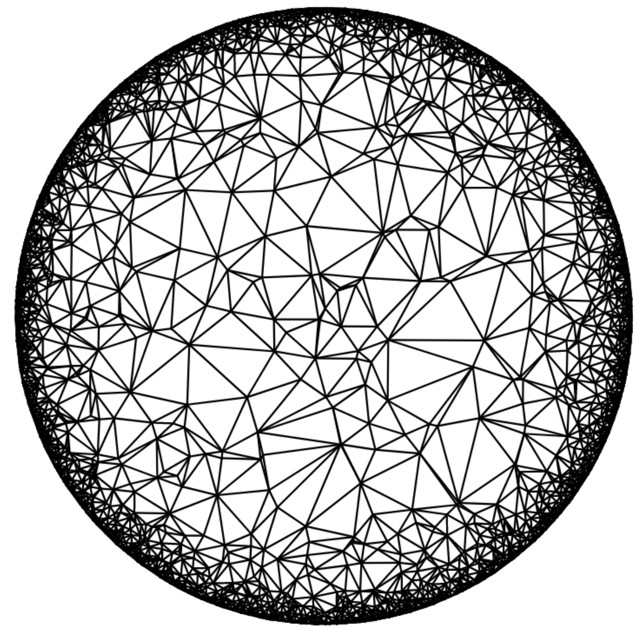
Question : Can the PSHITs be interpreted as local limits ?

A conjecture

Let $\frac{g_n}{n} \to \theta$ with $\theta \in [0, \frac{1}{2}[$.
Let (T_n) be a sequence of random triangulations, such that T_n is drawn uniformly among all triangulations of genus g_n with $2n$ triangles.

Conjecture [Benjamini, Curien '12] : (T_n) has a local limit, and it is a PSHIT of parameter λ, with λ a function of θ.

For g_n constant, the limit is the UIPT (well known,but never written anywhere).



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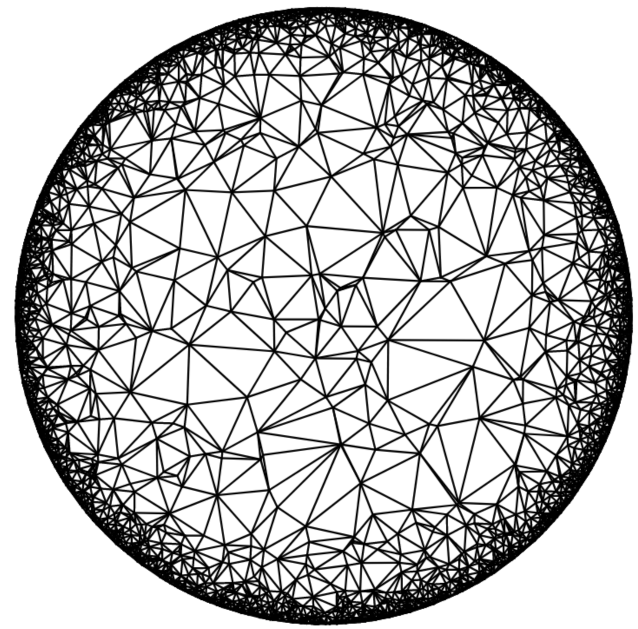

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A similar result [Angel, Chapuy, Curien, Ray '13] : the local limit of one-faced maps of high genus is an infinite hyperbolic tree

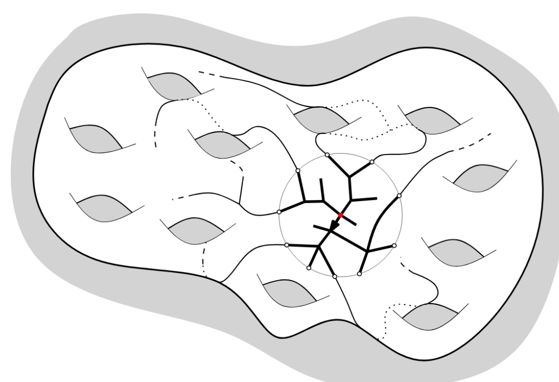
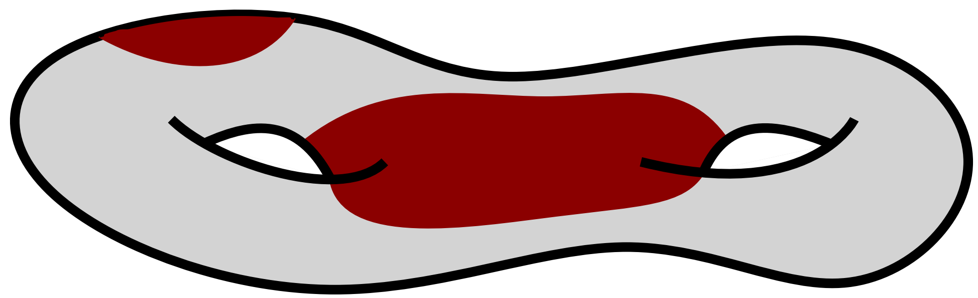


image : ACCR

The intuition behind the conjecture

For fixed genus . . .



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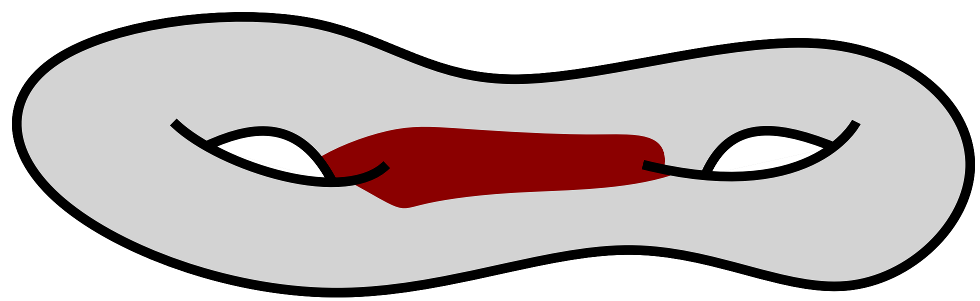

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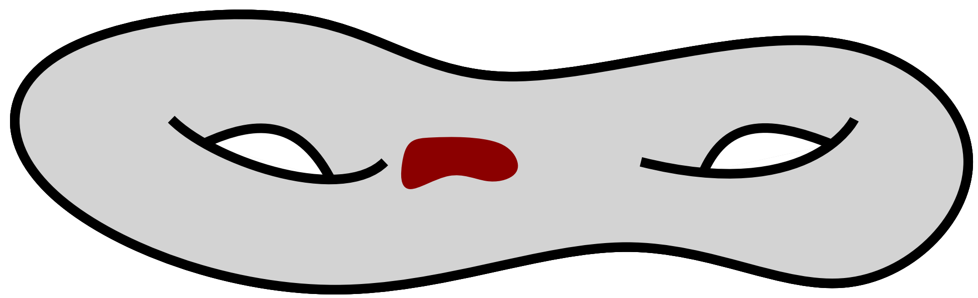

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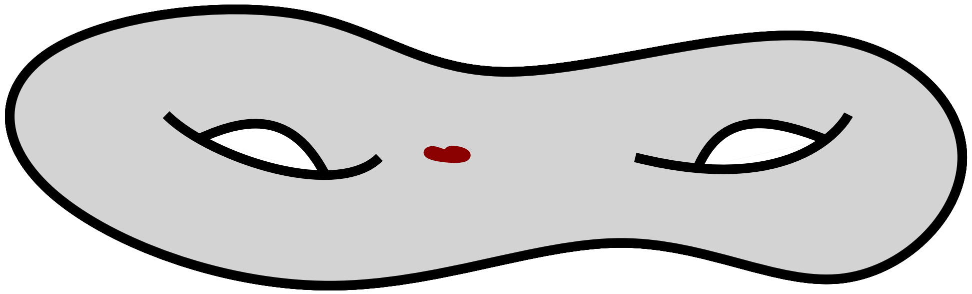

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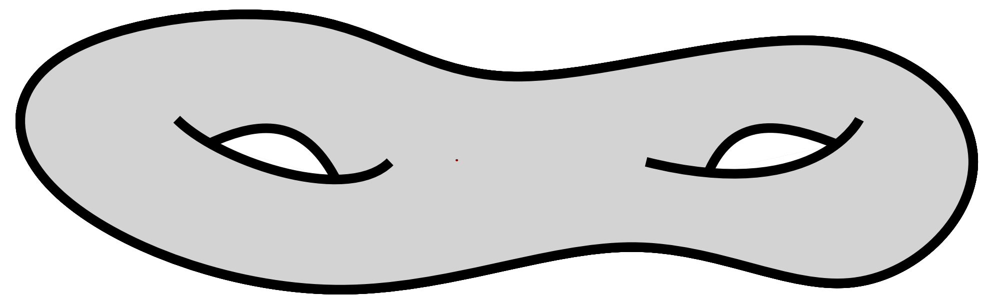

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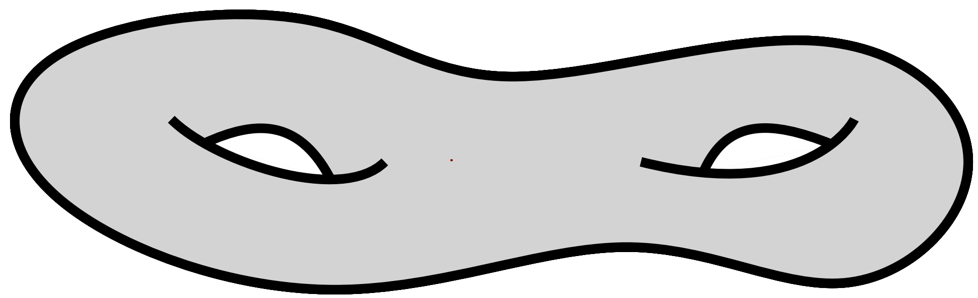

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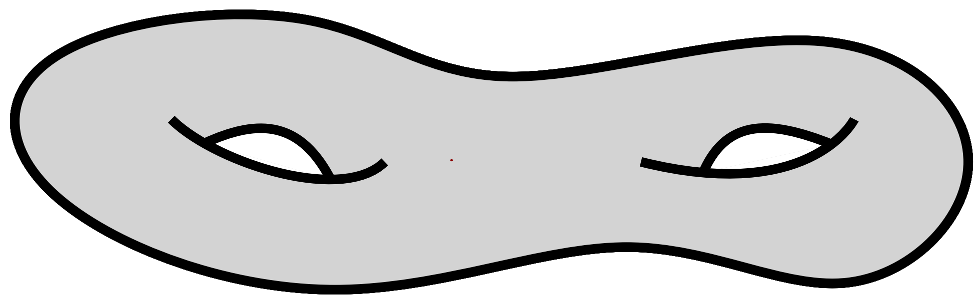

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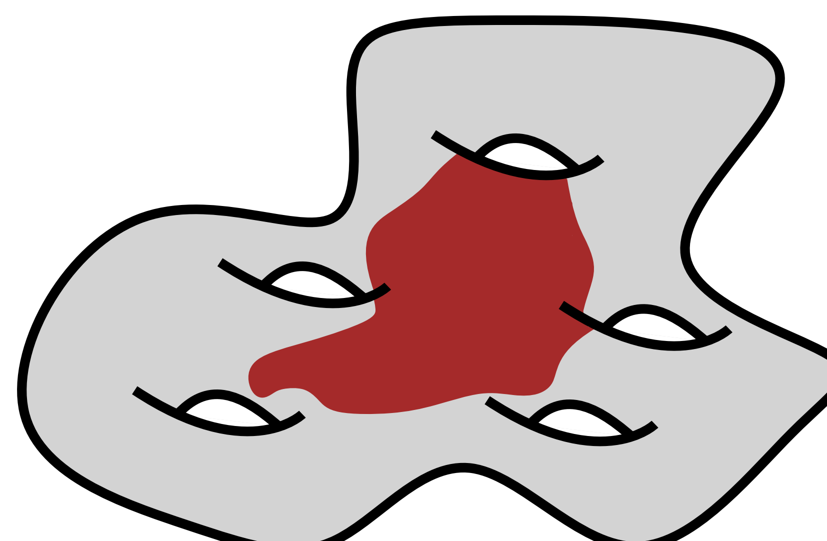

When the genus increases linearly with the size, in the end we don't see the genus but we still "feel the curvature"

Our result

Theorem [Budzinski, L. '18+] : the conjecture of Benjamini and Curien is true.

First idea :

Obtain precise asymptotics for $\tau(n,g)$ (the number of triangulations of genus g with $2n$ triangles) as $\frac{g}{n} \to \theta$

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$\downarrow$

TOO HARD

Outline of the proof :

1) Tightness (+ planarity and one-endedness)
$\rightarrow$ every subsequence has a converging subsubsequence

2) Every possible limit is a PSHIT with random parameter Λ

3) Λ is deterministic and depends only on θ

Bonus : asymptotics !

$$\frac{\tau(n, g_n)}{\tau(n-1, g_n)} \to c(\theta)$$

$$\tau(n, g_n) = n^{2g_n} \exp(nf(\theta) + o(n))$$

What's next ?

Boltzmann maps

Diameter of high genus maps ($= \log n$, [Chapuy, L., Marzouk '19+])

Maps decorated with "matter" ?

What happens when $\frac{g}{n} \to \frac{1}{2}$?

More geometric info on high genus maps ?

Thank you !